Polynomial Zeros (1.5) HW

1. The polynomial f is given by $f(x) = x^3 - 8x^2 + 16x$
   a. How many zeros does f have?
   b. How many distinct real zeros does f have?
   c. How many non-real zeros does f have?
   d. How many x-intercepts does the graph of f have?
   e. At which x-intercepts (if any) does the graph of f cross the x-axis?
   f. At which x-intercepts (if any) is the graph of f tangent to the x-axis?

2. The polynomial g is given by $g(x) = (x+2)^2(x^2+2)$
   a. How many zeros does f have?
   b. How many distinct real zeros does f have?
   c. How many non-real zeros does f have?
   d. How many x-intercepts does the graph of f have?
   e. At which x-intercepts (if any) does the graph of f cross the x-axis?
   f. At which x-intercepts (if any) is the graph of f tangent to the x-axis?

3. The polynomial h is given by $h(x) = (x^2-9)(2x^2+3x-9)$
   a. How many zeros does f have?
   b. How many distinct real zeros does f have?
   c. How many non-real zeros does f have?
   d. How many x-intercepts does the graph of f have?
   e. At which x-intercepts (if any) does the graph of f cross the x-axis?
   f. At which x-intercepts (if any) is the graph of f tangent to the x-axis?

4. Find the y-intercept of $y = 3(x-2)^2(x+3)(x-1)$

5. The polynomial function p is given by $p(x) = -\frac{1}{3}(x-1)(x^2+4x-5)^2$. Which one of the following statements is false?
   (A) $\lim_{x\to-\infty} p(x) = \infty$
   (B) $\lim_{x\to\infty} p(x) = -\infty$
   (C) p has exactly two distinct real zeros
   (D) p has exactly one relative maximum and zero relative minima.

6. The polynomial r is given by $r(x) = (2x^2 - 6x + 5)(2x^2 - 11x + 5)$. Which one of the following statements is true?
   (A) r has four distinct real zeros
   (B) r has two distinct real zeros and no non-real zeros
   (C) r has no real zeros and two distinct non-real zeros
   (D) r has two distinct real zeros and two distinct non-real zeros

7. The polynomial f is given by $f(x) = -5(x-5)^2(x+1)$. Which one of the following must be true?
   (A) Between $x = -1$ and $x = 5$ there must be an input value corresponding to a local maximum of f.
   (B) Between $x = -1$ and $x = 5$ there must be an input value corresponding to a local minimum of f.
   (C) At $x = 5$, f has a local minimum.
   (D) As $x = -1$, f has a local maximum.

8. Construct a polynomial for which 1 is a zero of multiplicity 2. Make your polynomial have leading coefficient 1 and use the smallest possible degree for your polynomial. Write your polynomial in standard form.

9. Construct a third-degree polynomial for which $f(-2) = 0$, $f(1) = 0$, $f(2) = 0$, and $f(3) = 20$.

10. Find all zeros of $f(x) = (x^2 - 6x + 13)(4x^2 - 4x + 1)$. For each zero, determine the multiplicity.

11. $f(x)$ is a polynomial with the following properties:
    - The graph of $f(x)$ is tangent to the x-axis at $x = 5$
    - $f(3) = 0$
    - $\frac{2}{3} + i$ is a zero of f

    What is the smallest possible degree of f?

12. $f(x)$ is a polynomial with the following properties:
    - $(x - 2)$ is a factor of $f(x)$
    - $f(0) = 0$
    - As input values of $f(x)$ decrease without bound, the output values increase without bound.
    - As input values of $f(x)$ increase without bound, the output values decrease without bound.

    What is the smallest possible degree of f?

Each of the following questions describes a polynomial function. However, each scenario is impossible. Give a reason why the proposed polynomial is impossible.

13. $f(1) = 0$, $f(4) = 0$, f has no relative extrema, $\lim_{x\to\infty} f(x) = -\infty$

14. All zeros of f have an even multiplicity, $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$, the graph of $f(x)$ has three x-intercepts

Polynomial Zeros (1.5) HW Answers

1.
   a. 3
   b. 2
   c. 0
   d. 2
   e. $(0,0)$
   f. $(4,0)$
2.
   a. 4
   b. 1
   c. 2
   d. 1
   e. None
   f. $(-2,0)$
3.
   a. 4
   b. 3
   c. 0
   d. 3
   e. $\left(\frac{3}{2},0\right)$ and $(3,0)$
   f. $(-3,0)$
4. $(0,-36)$
5. D
6. D
7. B
8. $f(x)=x^2-2x+1$
9. $f(x)=2(x+2)(x-1)(x-2)$ or $f(x)=2x^3-2x^2-8x+8$
10. $3+2i$ (multiplicity 1)
    $3-2i$ (multiplicity 1)
    $\frac{1}{2}$ (multiplicity 2)
11. 5 (AKA quintic)
12. 3
13. Between two real zeros of a polynomial, there must a relative extremum. If $f(1)=0$ and $f(4)=0$, somewhere between 1 and 4 is at least one x-value corresponding with a relative extremum.
14. The degree of a polynomial will always match the sum of the multiplicities of its zeros. If all the zeros are of an even multiplicity, the polynomial will be of an even degree. So, the left end behavior and right end behavior should match. This contradicts the limits given.